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AN ALGORITHM OF DETERMINATION OF FEASIBLE FORCES AND MOMENTS FOR MULTIPROPELLER UNDERWATER ROBOT

DAUGIAPROPELERINIAME POVANDENINIAME ROBOTE VEIKIANČIŲ JĖGŲ IR MOMENTŲ NUSTATYMO ALGORITMAS

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Power distribution in a propulsion system is a key problem in control synthesis of a multipropeller underwater robotic vehicle. Computation of thrusts from propelling forces and moments is a model based optimisation problem that in the simplest form is unconstrained. However, in practice where physical limitations must be taken into account, obtained in such a way solution may be unrealised. On the basis of the analytic geometry, taking into account restrictions put on the propelling forces and moments, a method for evaluation of ability of the propulsion system to produce demanded commands is described. In case of lack of such ability, an algorithm of modification of the commands and determination of the feasible forces and moments is presented. Due to computational simplicity the proposed approach seems to be a good solution in real-time applications.

Underwater robot, hydrodynamic thrust allocation, propulsion system.

Introduction

Nowadays, it is common to use underwater robotic vehicles (URVs) to accomplish such missions as inspection of coastal and off-shore structures, cable maintenance, as well as hydrographical and biological surveys. In the military field they are employed in such tasks as surveillance, intelligence gathering, torpedo recovery and mine counter measures.

Their motion of six degrees of freedom (DOF) can be described by the following vectors [1, 2, 3]:

$$\begin{aligned} \boldsymbol{\eta} &= [x, y, z, \phi, \theta, \psi]^T \\ \mathbf{v} &= [u, v, w, p, q, r]^T \\ \boldsymbol{\tau} &= [X, Y, Z, K, M, N]^T \end{aligned} \tag{1}$$

where:

$\boldsymbol{\eta}$ – vector of position and orientation in the inertial frame;

x, y, z – coordinates of position;

ϕ, θ, ψ – coordinates of orientation (Euler angles);

$\mathbf{v}$ – the linear and angular velocity vector with coordinates in the body-fixed frame;

u, v, w – linear velocities along longitudinal, transversal and vertical axes;

p, q, r – angular velocities about longitudinal, transversal and vertical axes;

$\boldsymbol{\tau}$ – vector of forces and moments acting on the robot in the body-fixed frame;

X, Y, Z – forces along longitudinal, transversal and vertical axes;

K, M, N – moments about longitudinal, transversal and vertical axes.

The modern URVs are more and more frequently equipped with an automatic control system in order to execute complex manoeuvres without constant human intervention. Basic modules of the control system are depicted in Fig. 1. An autopilot computes demanded propelling forces and moments (commands) $\boldsymbol{\tau}_d$ by comparing desired position and orientation of the robot with their current estimates. Corresponding with them propeller thrusts **f** are calculated in a thrust distribution module and transmitted as control inputs to a propulsion system.

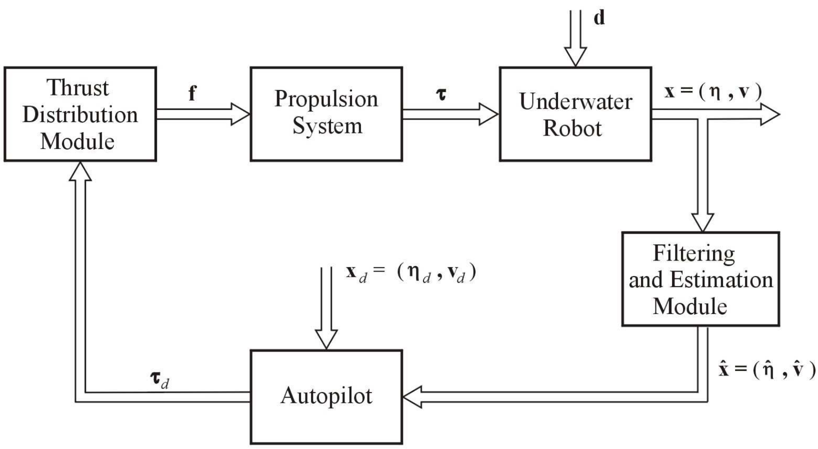

Fig. 1. Structure of the control system (**d** – a vector of environmental disturbances)
1 pav. Valdymo sistemos struktūra (**d** – aplinkos trikdymų vektorius)

Both movement and positioning of the underwater robot is realised only by change of propeller thrusts that leads to the change of propelling forces and moments. Control laws implemented in the autopilot are of general character and usually do not take into account constraints put on maximum and minimum values of thrusts developed by the propellers. It may cause that desired solution $\boldsymbol{\tau}_d$ cannot be realised by the propulsion system due to work of one or more thrusters in saturation. Such a situation can make a contribution to deterioration of control and behaviour of the robot may differ from a required one significantly.

Therefore, thrust distribution is one of tasks of the control system that has an essential influence on quality of control. A procedure of thrust allocation is proposed to be realised in two stages (Fig. 2). In the first stage a generating capacity of the demanded commands $\boldsymbol{\tau}_d$ by the propulsion system is checked and feasible commands $\boldsymbol{\tau}'_d$ are determined, (i.e. such values of forces and moments which the propulsion system can produce). In the second one the real allocation of thrusts among the propellers is carried out on the basis of $\boldsymbol{\tau}'_d$.

Procedure of thrust allocation for horizontal motion

For the conventional URVs basic motion is movement in a horizontal plane with some variation due to diving. They operate in crab-wise manner in four DOF with small roll and pitch angles that can be neglected during normal operations. Therefore, their spatial motion is regarded as superposition of two displacements: motion in the horizontal plane and motion in the vertical plane. It allows dividing the propulsion system into two independent subsystems responsible for movement in the vertical and horizontal planes, consequently. A general structure of such a system shows Fig. 3.

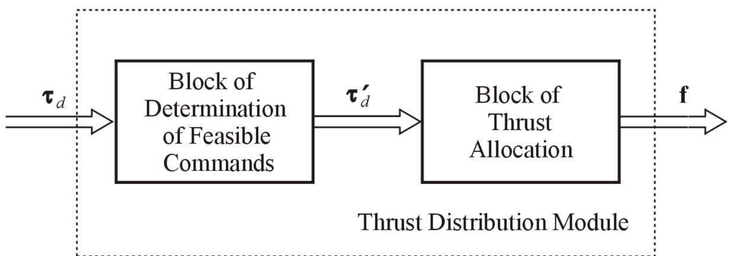

Fig. 2. A block diagram of the thrust distribution module
2 pav. Traukos paskirstymo modulio blokinė diagrama

The first subsystem enables motion in heave and consists of two thrusters. The demanded force Z_d is equal to a sum of their thrusts.

The second one assures motion in surge, sway and yaw and composes of four thrusters mounted askew in relation to main vehicle’s symmetry axes (see Fig.

4). Hence, desired forces X_d and Y_d acting in the longitudinal and transversal axes and moment N_d about the vertical axis are combination of thrusts produced by the thrusters.

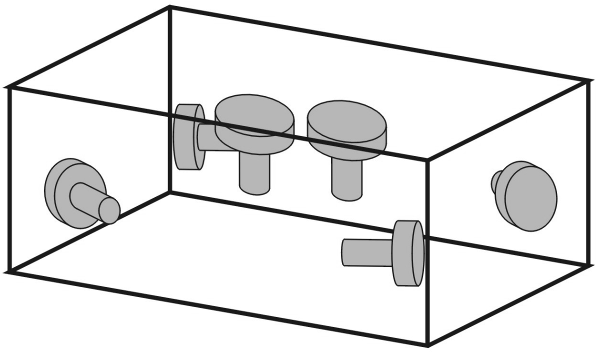

Fig. 3. A structure of propulsion system with 6 thrusters
3 pav. 6 stūmiklių varymo į priekį sistemos struktūra

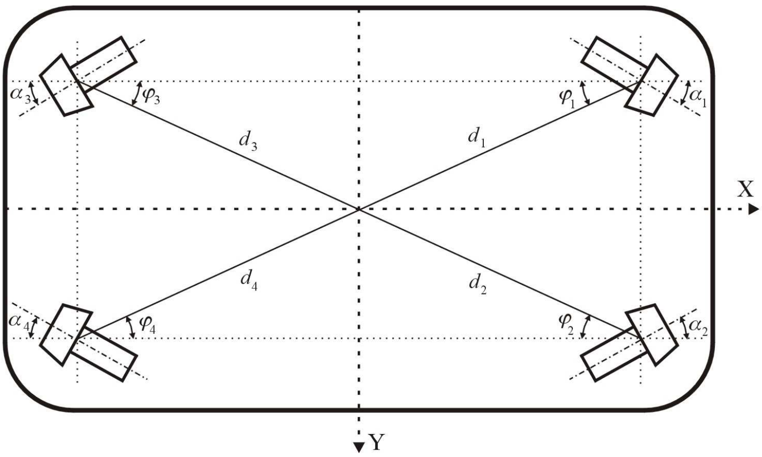

Fig. 4. Layout of thrusters in subsystem responsible for horizontal motion
4 pav. Horizontalaus judesio posistemės stūmiklių įrengimo schema

Let us denote:

$\boldsymbol{\tau}_d = [\tau_{d1}, \tau_{d2}, \tau_{d3}]^T = [X_d, Y_d, N_d]^T$ - a vector of demanded commands,

$\mathbf{f} = [f_1, f_2, ..., f_4]^T$ - a thrust vector,

and assume that components of the both vectors are bounded:

$$\tau_{di}^2 - \left(\tau_i^{\max}\right)^2 \leq 0 \qquad \text{for} \quad i = \overline{1,3} \tag{2}$$

$$f_j^2 - \left(f_j^{\max}\right)^2 \le 0 \quad \text{for} \quad j = \overline{1,4} \tag{3}$$

Values of $\tau_i^{\max}$ and $f_j^{\max}$ depend on propellers design and a configuration of thrusters in the propulsion system.

As shown in [1, 5] for the horizontal motion the vector of demanded propelling forces and moment $\boldsymbol{\tau}_d$ can be described as a function of the thrust vector **f** by the following expression:

$$\boldsymbol{\tau}_d = \mathbf{T}(\boldsymbol{\alpha})\mathbf{f} \tag{4}$$

where:

$\mathbf{T}(\boldsymbol{\alpha})$ – thrusters configuration matrix:

$$\mathbf{T}(\boldsymbol{\alpha}) = \begin{bmatrix} \mathbf{t}_1 \\ \mathbf{t}_2 \\ \mathbf{t}_3 \end{bmatrix} = \begin{bmatrix} \cos(\alpha_1) & \cos(\alpha_2) & \dots & \cos(\alpha_4) \\ \sin(\alpha_1) & \sin(\alpha_2) & \dots & \sin(\alpha_4) \\ d_1 \sin(\alpha_1 - \varphi_1) & d_2 \sin(\alpha_2 - \varphi_2) & \dots & d_4 \sin(\alpha_4 - \varphi_4) \end{bmatrix},$$

$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots, \alpha_4]^T$ – vector of azimuth angles,

α_i – angle between longitudinal axis and direction of thrust of the i^{th} propeller f_i,

d_i – distance of the i^{th} thruster from a centre of gravity,

φ_i – angle between longitudinal axis and the line connecting the centre of gravity with the i^{th} thruster symmetry centre.

The thrust allocation problem, i.e. computation of **f** from $\boldsymbol{\tau}_d$, is usually formulated as the least-squares optimisation problem and described in the following form [2, 4]:

$$\mathbf{f} = \mathbf{T}^*(\boldsymbol{\alpha})\boldsymbol{\tau}_d \tag{5}$$

where the matrix $\mathbf{T}^*(\boldsymbol{\alpha}) = \mathbf{T}^T(\boldsymbol{\alpha})\left(\mathbf{T}(\boldsymbol{\alpha})\mathbf{T}^T(\boldsymbol{\alpha})\right)^{-1}$ is the generalized inverse.

This method of thrust allocation allows to find a minimum-norm solution but it should be noted that (5) belongs to unconstrained optimisation problems – i.e., there are no bounds on elements of the vector **f**, so the obtained values f_i may not satisfy (3) and then generation of the desired vector $\boldsymbol{\tau}_d$ by the propulsion system is not possible. In such a case a new vector of commands meeting the condition (3) must be determined. A method of evaluation of this vector, called the vector of feasible commands and denoted by $\boldsymbol{\tau}'_d = [\tau'_{d1}, \tau'_{d2}, \tau'_{d3}]^T$, is presented in the next section.

An algorithm of determination of feasible propelling forces and moment

Assume that the propulsion system consists of nonrotational thrusters. It means that quantities like: d_i, α_i and φ_i are constant for the every i^{th} thrusters. Hence, all elements of the configuration matrix $\mathbf{T}(\boldsymbol{\alpha})$ are constant.

Let us denote:

1. $\tau_1^{\max}$, $\tau_2^{\max}$ and $\tau_3^{\max}$ – maximum values of propelling forces and moment generated by the propulsion system for the horizontal motion:

$$\begin{aligned} \tau_1^{\max} &= \sum_{i=1}^{n} \left|\tau_{1i}^{\max}\right| = \sum_{i=1}^{n} \left| f_i^{\max} \cos(\alpha_i) \right| \\ \tau_2^{\max} &= \sum_{i=1}^{n} \left|\tau_{2i}^{\max}\right| = \sum_{i=1}^{n} \left| f_i^{\max} \sin(\alpha_i) \right| \\ \tau_3^{\max} &= \sum_{i=1}^{n} \left|\tau_{3i}^{\max}\right| = \sum_{i=1}^{n} \left| f_i^{\max} d_i \sin(\alpha_i - \varphi_i) \right| \end{aligned} ,$$

2. O – an origin of the Cartesian coordinate system,
3. P – a point in the 3-dimensional space with coordinates $(\tau_{d1}, \tau_{d2}, \tau_{d3})$,
4. $\overrightarrow{OP}$ – a position vector of the point P.

Evaluation of capacity of the propulsion system to generation of the desired propelling forces and moment $\boldsymbol{\tau}_d$ requires taking into consideration both limitations (2) and (3) simultaneously.

The first one indicates that the vector $\boldsymbol{\tau}_d$ is produced only if the position vector $\overrightarrow{OP}$ is entirely contained in a cubicoid having vertexes in points: $(\tau_1^{\max}, \tau_2^{\max}, \tau_3^{\max})$, $(\tau_1^{\max}, \tau_2^{\max}, -\tau_3^{\max})$, $(\tau_1^{\max}, -\tau_2^{\max}, \tau_3^{\max})$, $(\tau_1^{\max}, -\tau_2^{\max}, -\tau_3^{\max})$, $(-\tau_1^{\max}, \tau_2^{\max}, \tau_3^{\max})$, $(-\tau_1^{\max}, \tau_2^{\max}, -\tau_3^{\max})$, $(-\tau_1^{\max}, -\tau_2^{\max}, \tau_3^{\max})$, $(-\tau_1^{\max}, -\tau_2^{\max}, -\tau_3^{\max})$ (see Fig. 5). Since the components of the vector of demanded commands $\boldsymbol{\tau}_d$ are a linear combination of thrusts developed by all propellers then fulfilling only the condition (2) is not a guarantee their generation. E.g. if to any element of the vector $\boldsymbol{\tau}_d$ there corresponds an assignment $\tau_{di} = \tau_i^{\max}$, then full power of the propulsion system is used to its production and the rest of the components must be equal to zero. Therefore evaluation of capacity of the propulsion system to generation of the vector $\boldsymbol{\tau}_d$ requires giving either consideration to the inequality (3).

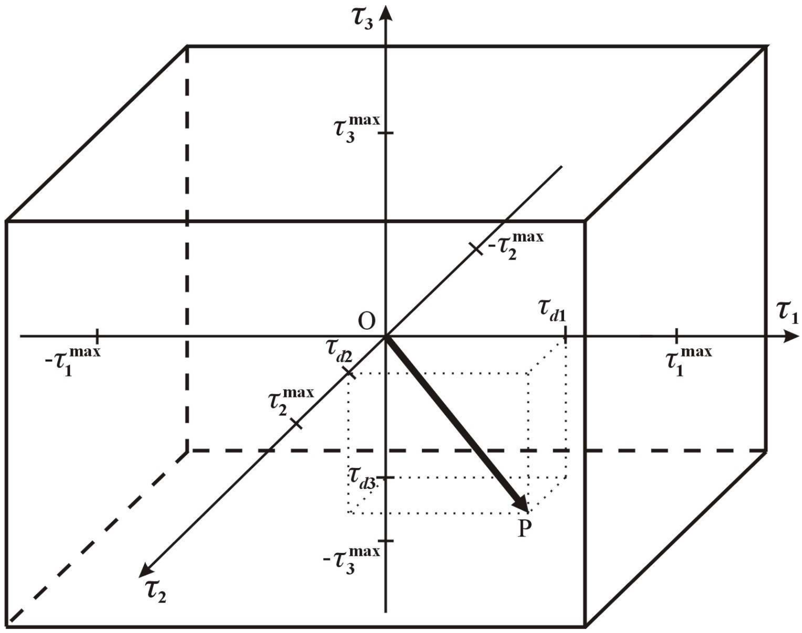

Fig. 5. A view of the cubicoid and the position of vector $\overrightarrow{OP}$
5 pav. Kubinis jėgų ir momentų išdėstymo vaizdas ir vektoriaus $\overrightarrow{OP}$ padėtis

Analysis of values that elements of the vector $\boldsymbol{\tau}_d$ may take under limitations (2) and (3) leads to the following conclusion: quantities τ_{d1}, τ_{d2} and τ_{d3} can be produced by the propulsion system if and only if the position vector $\overrightarrow{OP}$ is entirely contained in a trisoctahedron with vertexes in points: $(\tau_1^{max},0,0)$, $(0,\tau_2^{max},0)$, $(0,0,\tau_3^{max})$, $(-\tau_1^{max},0,0)$, $(0,-\tau_2^{max},0)$, $(0,0,-\tau_3^{max})$ (see Fig. 6). This situation proceeds if the following inequality holds:

$$\frac{|\tau_{d1}|}{\tau_1^{max}}+\frac{|\tau_{d2}|}{\tau_2^{max}}+\frac{|\tau_{d3}|}{\tau_3^{max}}\leq 1 \tag{6}$$

If (6) is false then the point $\mathrm{P}=(\tau_{d1},\tau_{d2},\tau_{d3})$ lies outside of the octahedron and the vector $\boldsymbol{\tau}_d$ can not be generated. It means that the vector of feasible commands $\boldsymbol{\tau}'_d=[\tau'_{d1},\tau'_{d2},\tau'_{d3}]^T$ must be determined. Its elements, under assumption that a reciprocal ratios of corresponding themselves components of the vectors $\boldsymbol{\tau}_d$ and $\boldsymbol{\tau}'_d$ are preserved:

$$\frac{\tau'_{d1}}{\tau'_{d2}} = \frac{\tau_{d1}}{\tau_{d2}} \text{ and } \frac{\tau'_{d1}}{\tau'_{d3}} = \frac{\tau_{d1}}{\tau_{d3}} \tag{7}$$

can be computed by means of the following equations:

$$\begin{aligned} \tau'_{d1} &= sign(\tau_{d1})\left(\frac{1}{\tau_1^{\max}} + \frac{1}{\tau_2^{\max}}\left|\frac{\tau_{d2}}{\tau_{d1}}\right| + \frac{1}{\tau_3^{\max}}\left|\frac{\tau_{d3}}{\tau_{d1}}\right|\right)^{-1} \\ \tau'_{d2} &= sign(\tau_{d2})\left|\frac{\tau_{d2}}{\tau_{d1}}\tau'_{d1}\right| \\ \tau'_{d3} &= sign(\tau_{d3})\left|\frac{\tau_{d3}}{\tau_{d1}}\tau'_{d1}\right| \end{aligned} \tag{8}$$

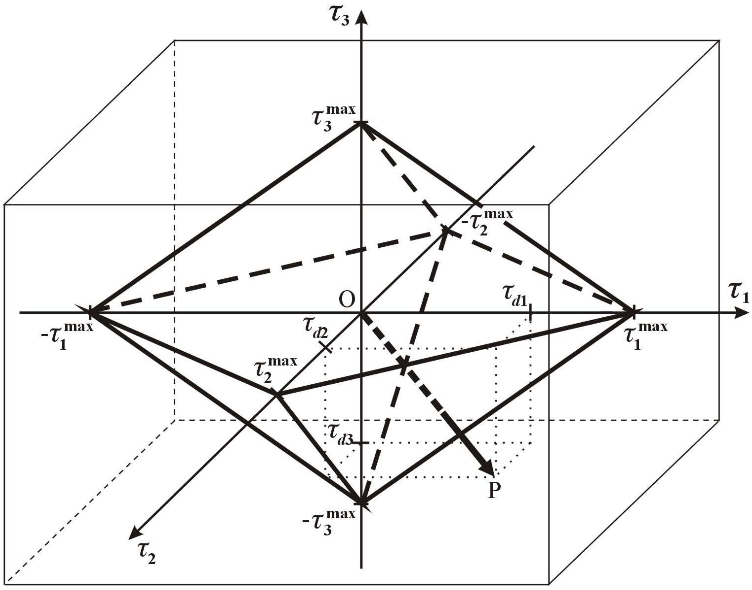

Fig. 6. A view of the trisoctahedron and the position of vector $\overrightarrow{OP}$.

6 pav. Trisoctahedronis jėgų ir momentų išdėstymas ir vektoriaus $\overrightarrow{OP}$ padėtis

Basis on the above considerations an algorithm of evaluation of the vector $\boldsymbol{\tau}_d$ and determination of $\boldsymbol{\tau}'_d$ has been designed (see Fig. 7). Input data to the algorithm are quantities $\tau_1^{\max}$, $\tau_2^{\max}$, $\tau_3^{\max}$ and the vector $\boldsymbol{\tau}_d$. The vector of feasible commands $\boldsymbol{\tau}'_d$ is computed according to (8).

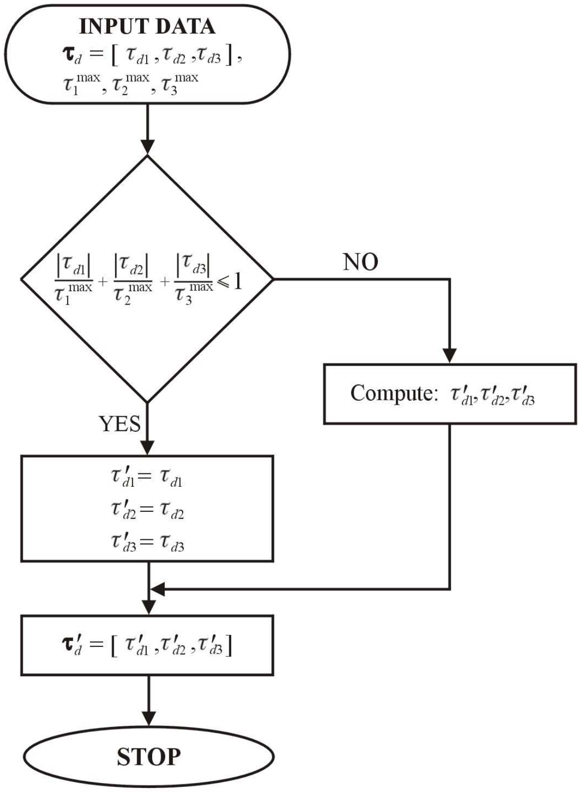

Fig. 7. A block diagram of the algorithm of determination of feasible commands
7 pav. Algoritmo blokinė diagrama veikiančioms komandoms nustatyti

A proof of the dependence (8)

Not to decreasing of generality of considerations it is assumed that $\tau_{di} \geq 0$ for $i = \overline{1,3}$. Such approach allows the analysis to restrict into a subspace limited by positive semi-axes of the coordinate system (see Fig. 8).

Let $A = (\tau_1^{max}, 0, 0)$, $B = (0, \tau_2^{max}, 0)$, $C = (0, 0, \tau_3^{max})$, $P = (\tau_{d1}, \tau_{d2}, \tau_{d3})$ and $P' = (\tau'_{d1}, \tau'_{d2}, \tau'_{d3})$ be points in the 3-dimensional space. An equation of a plane including the points A, B and C has a form:

$$\frac{\tau_1}{\tau_1^{max}} + \frac{\tau_2}{\tau_2^{max}} + \frac{\tau_3}{\tau_3^{max}} = 1 \quad (9)$$

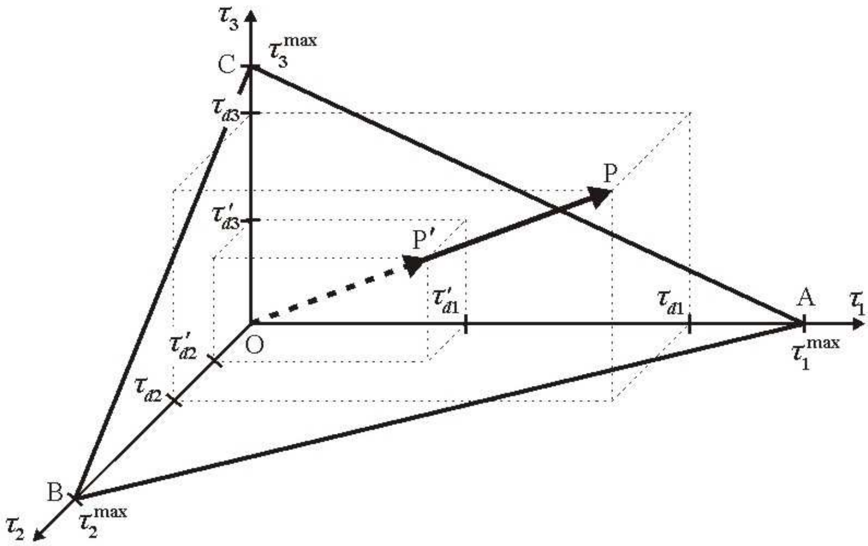

Fig. 8. A view of position vectors $\overrightarrow{OP}$ and $\overrightarrow{OP'}$ for positive semi-axes of the Cartesian coordinate system

8 pav. Padėties vektorių $\overrightarrow{OP}$ ir $\overrightarrow{OP'}$ vaizdas teigiamuose Cartesian koordinačių sistemos pusašiuose

Let us assume that the point $P' = (\tau'_{d1}, \tau'_{d2}, \tau'_{d3})$ is a common point of a line containing the position vector $\overrightarrow{OP}$ and the plane defined by (9). Substituting the coordinates of the point P' into (9) and taking into account the requirements (7) the following set of equations is formulated:

$$\begin{cases} \dfrac{\tau'_{d1}}{\tau_1^{\max}} + \dfrac{\tau'_{d2}}{\tau_2^{\max}} + \dfrac{\tau'_{d3}}{\tau_3^{\max}} = 1 \\ \dfrac{\tau'_{d1}}{\tau'_{d2}} = \dfrac{\tau_{d1}}{\tau_{d2}} \\ \dfrac{\tau'_{d1}}{\tau'_{d3}} = \dfrac{\tau_{d1}}{\tau_{d3}} \end{cases} \tag{10}$$

Hence, solving (10) the following expressions for calculation of τ'_{d1}, τ'_{d2} and τ'_{d3} are obtained:

$$
\begin{aligned}
\tau'_{d1} &= \left(\frac{1}{\tau_1^{\max}} + \frac{1}{\tau_2^{\max}} \frac{\tau_{d2}}{\tau_{d1}} + \frac{1}{\tau_3^{\max}} \frac{\tau_{d3}}{\tau_{d1}} \right)^{-1} \\
\tau'_{d2} &= \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \\
\tau'_{d3} &= \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1}
\end{aligned}
\tag{11}
$$

Finally, transformation of the above dependences to the entirely Cartesian coordinate system yields:

$$
\begin{aligned}
\tau'_{d1} &= sign(\tau_{d1}) \left(\frac{1}{\tau_1^{\max}} + \frac{1}{\tau_2^{\max}} \left| \frac{\tau_{d2}}{\tau_{d1}} \right| + \frac{1}{\tau_3^{\max}} \left| \frac{\tau_{d3}}{\tau_{d1}} \right| \right)^{-1} \\
\tau'_{d2} &= sign(\tau_{d2}) \left| \frac{\tau_{d2}}{\tau_{d1}} \tau'_{d1} \right| \\
\tau'_{d3} &= sign(\tau_{d3}) \left| \frac{\tau_{d3}}{\tau_{d1}} \tau'_{d1} \right|
\end{aligned}
\tag{12}
$$

<u>End of prove.</u>

Conclusions

The paper presents a method of determination of feasible propelling forces and moment for underwater robotic vehicles. For the robot moving in the horizontal plane it is necessary to distribute the propelling forces and moment $\boldsymbol{\tau}_d \in \Re^3$ among *n* propellers in terms of thrust $\mathbf{f} \in \Re^n$. Computation of **f** from $\boldsymbol{\tau}_d$ is a model based optimisation problem that in the simplest form is unconstrained.

However in real applications due to physical limitations, (e.g. saturation), this task must be solved as a constrained optimisation problem. To cope with those difficulties a procedure of checking demanded forces and moment and determination of feasible ones has been worked out. It allows finding such a vector $\boldsymbol{\tau}'_d$ that the unconstrained optimisation methods can be used without any restrictions.

The main advantage of the approach is its simplicity and flexibility with regard to the construction of the vehicle’s power transmission system and number of thrusters. The developed procedure of determination of feasible commands is of general character and can be successfully applied to all types of the URVs.

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Ержи Гарус

АЛГОРИТМ ОПРЕДЕЛЕНИЯ ДЕЙСТВУЮЩИХ СИЛ И МОМЕНТОВ В МУЛЬТИПРОПЕЛЛЕРНОМ ПОДВОДНОМ РОБОТЕ

Аннотация

Распределение мощности в системе движителя является основной проблемой управления синтеза многопропеллерного подводного транспортного робота. Расчет тяги по ведущим силам и моментам оказывается оптимизационной проблемой базирующейся на моделировании, которая в натуральной форме не является ограниченной. Однако, на практике, когда ряд физических ограничений необходимо принять во внимание, указанным путем получаемое решение может оказаться нереализуемой. С использованием теории аналитической геометрии и приняв во внимание накладываемые на движущие силы и моменты ограничения, в статье приводится метод для оценки возможности системы движителя производить (выдавать) необходимые команды управления. В случае недостаточной возможности, предлагается алгоритм модификации команд и определения действующих сил и моментов. В связи с простотой расчетов, предлагаемый метод может оказаться полезным для выполнения срочных оперативных действий.

Подводный робот, локализация гидродинамической тяги, система движителя.

Jerzy Garus

DAUGIAPROPELERINIAME POVANDENINIAME ROBOTE VEIKIANČIŲ JĖGŲ IR MOMENTŲ NUSTATYMO ALGORITMAS

Reziumė

Galios paskirstymas varymo (stūmimo) į priekį sistemoje yra pagrindinė daugiapropelerinio povandeninio transportinio roboto valdymo sintezės problema. Stūmimų į priekį apskaičiavimas pagal varančiąsias jėgas ir momentus yra modeliavimu grindžiama optimizavimo problema, kuri natūralia forma nėra suvaržyta. Tačiau, praktiškai, kai tenka priimti įvairius fizikinius apribojimus, tokiu būdu gaunamas sprendinys gali būti nerealus ir neįvykdomas. Pritaikius analitinės geometrijos teoriją ir atsižvelgus į varančiosioms jėgoms ir momentams taikomus apribojimus, straipsnyje aprašomas metodas kaip įvertinti varymo sistemos galimybę pagaminti reikiamas valdymo komandas. Esant nepakankamai galimybei, pateikiamas komandų modifikavimo ir galimų jėgų bei momentų nustatymo algoritmas. Apskaičiavimų paprastumo dėka, pateikiamas problemos sprendimo būdas gali būti naudingas skubiems operatyviems veiksmams atlikti.

Povandeninis robotas, hidrodinaminės traukos lokalizavimas, varymo/stūmimo sistema.